COLLEGE WB3413 DREDGING PROCESSES

The Breaching Process (September 2003)

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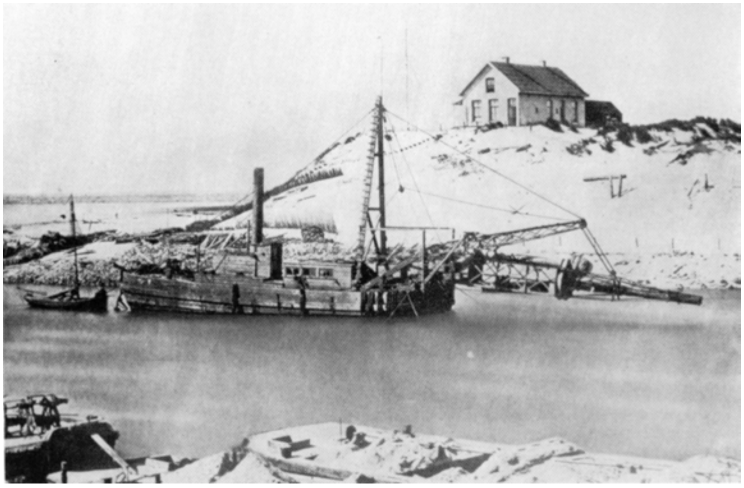
1. Intoduction

Breaching is the occurrence of continues and/or local instabilities on a sandy slope causing a density flow running downwards from the slope.
When dredging sand with a plain suction dredger this situation is purposely generated. This type of dredger, consisting in its most simple appearance of a pontoon, a suction tube, a dredge pump and a discharge pipeline, picks up the density current running from the slope.

The technique of dredging sand by means of a plain suction dredger is rather old. Already in 1872 when excavating the North Sea Channel from Amsterdam to the North Sea the dredging was done by a small wooden suction dredger (Sea figure). The hey days for this dredger was the period after the second World War till the eighties. For the large extension of the urban areas of Amsterdam en Rotterdam million cubic meters of sand was required from near by sources.

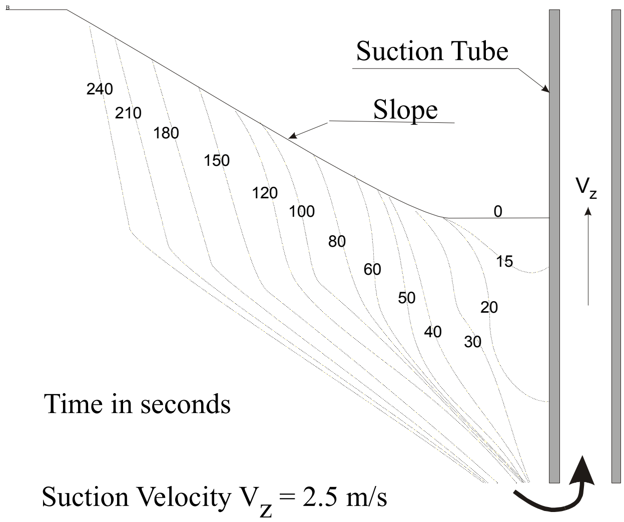


Wooden “Hutton” Dredger

Till today this techniques is used frequently for dredging sand fro reclamation projects as well as for the mining of sand and gravel for the concrete industry,

2. The breaching process

When a suction tube is lowered to a certain depth in a sand layer, a hole is created around the suction tube with almost vertical slopes. When time passes these vertical slopes move away radial from the suction tube (See Figure), while the sand flows over a certain slope to the suction mouth. These vertical moving slopes are called, depending on the scale of process, little walls (laboratory) or active banks (prototype).
The horizontal movements of these walls are independent of the movement of the suction tube. The sand flow causes a density current that might further erode the slope to the suction mouth.



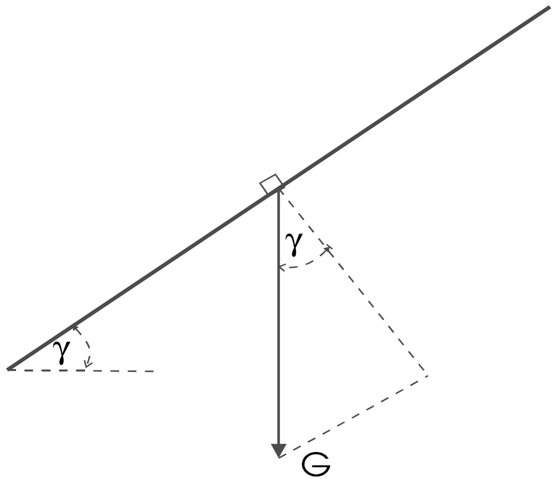
The stability of a sand particle on slope is comparable with a equilibrium of a block on a rough inclined plane under an angle β.
For stabilty:

$$G\sin\gamma \le f * G\cos\gamma$$

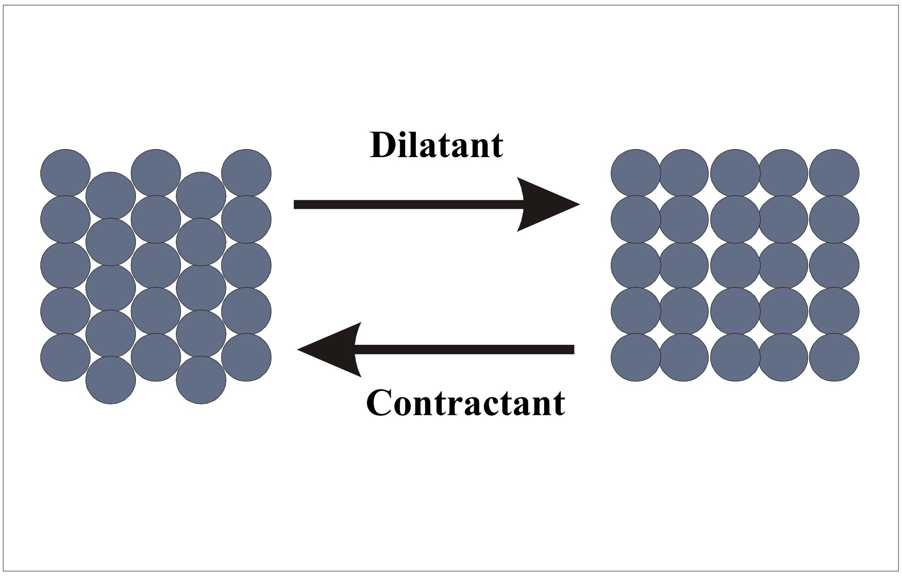
or

$$\tan\gamma \le f = \tan\phi$$

with ϕ the angle of internal friction.



However, a sand particle on a submerged slope is subjected to more forces than the gravity force F_2. For instant when groundwater flows out of the slope a seepage force F_1 is acting on the particle. Besides the natural groundwater flow, water may flow in or out of the slope by deformation of the particle matrix due to loads. When matrix of a dense sand is loaded by a shear force in such away that the particles moved over each other, then porosity will increase. The increase in pore volume has to be filled with water, which causes a additional flow. The increase in porosity is called dilatant behaviour. The opposite of it is called contractant behaviour and causes a outgoing flow.



The forces F_1 en F_2 worden beschouwd per eenheid van volume.

The particle is now stable when the following condition is met:

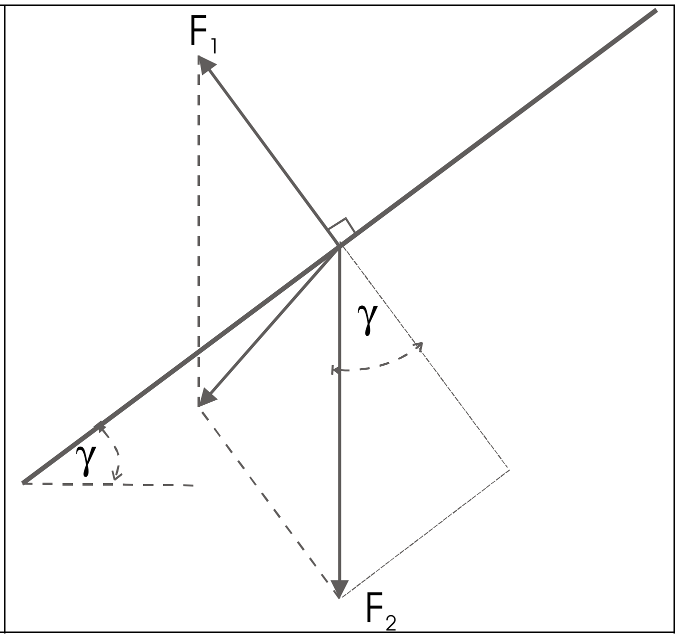
$$F_2 \sin\gamma \le \tan\phi * \left(F_2 \cos\gamma - F_1\right)$$

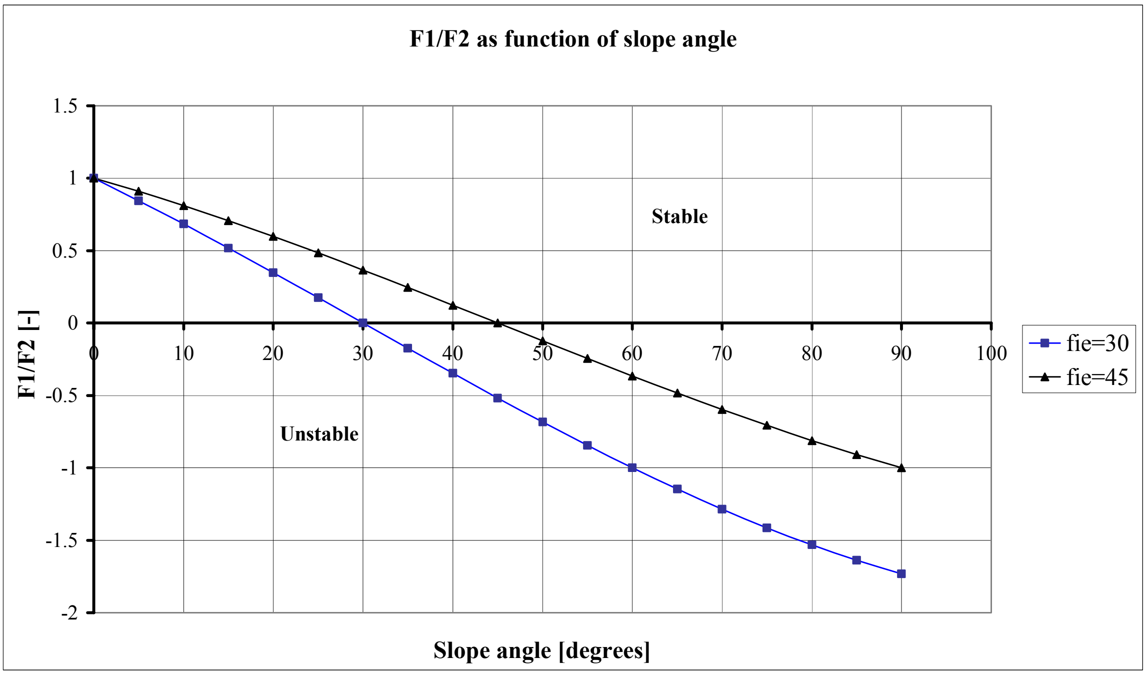
or

$$F_1 \ge F_2 \cos\gamma - \frac{F_2 \sin\gamma}{\tan\phi}$$

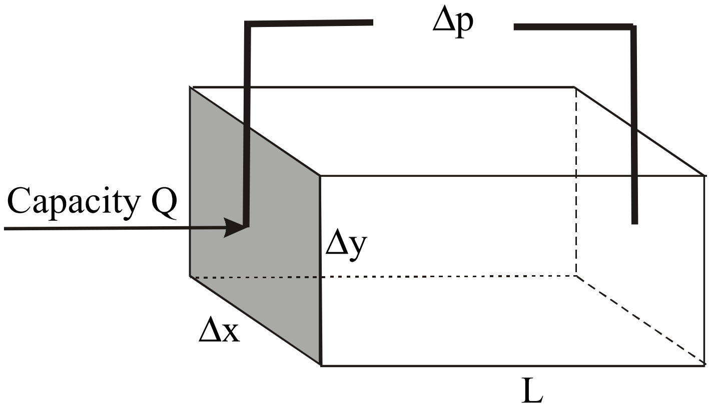
$$\frac{F_1}{F_2} \ge \cos\gamma - \frac{\sin\gamma}{\tan\phi}$$

The forces F_1 and F_2 are considered per unit of volume.





According to Darci's law is $v = k \cdot i = k \dfrac{\Delta p}{\rho_w g L}$ (Figure below)



In which:
v = discharge velocity [m/s]
k = permeability [m/s]
Δp=Pressure difference [Pa]
ρ_w=density water [kg/m^3]
g = gravity acceleration [m/s^2]
L = length over part considered [m]

The force F_1 (per unit of volume) is $\Delta p = \frac{F_1 \Delta x \Delta y L}{\Delta x \Delta y} \Rightarrow F_1 = \frac{\Delta p}{L}$

From $v = k \frac{\Delta p}{\rho_w g L}$ now follows: $F_1 = \frac{v}{k}\rho_w g$ and $F_2 = \left(\rho_k - \rho_w\right) g$ being the submerged weight of the particle.

Substituting these values in the stability condition, gives:

$$\frac{-v}{k} = \frac{\rho_k - \rho_w}{\rho_w}\left(\sin\gamma \cot\phi - \cos\gamma\right)$$

The discharge velocity *v* is related to the horizontal movements of the slope.

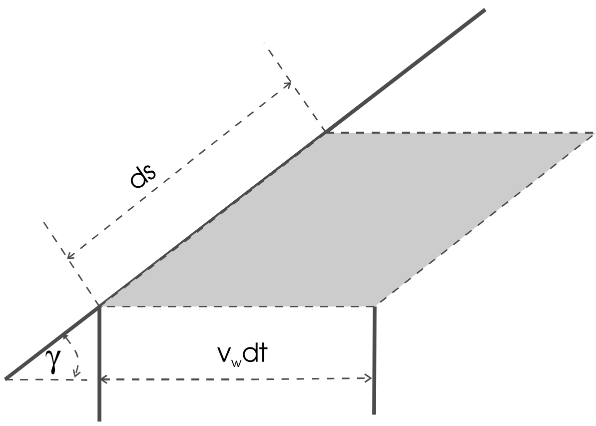
When the slopes moves with a horizontal speed v_w to the right and as a result the porosity increases with an amount of Δn, then the volume water that will enter the slope will be:

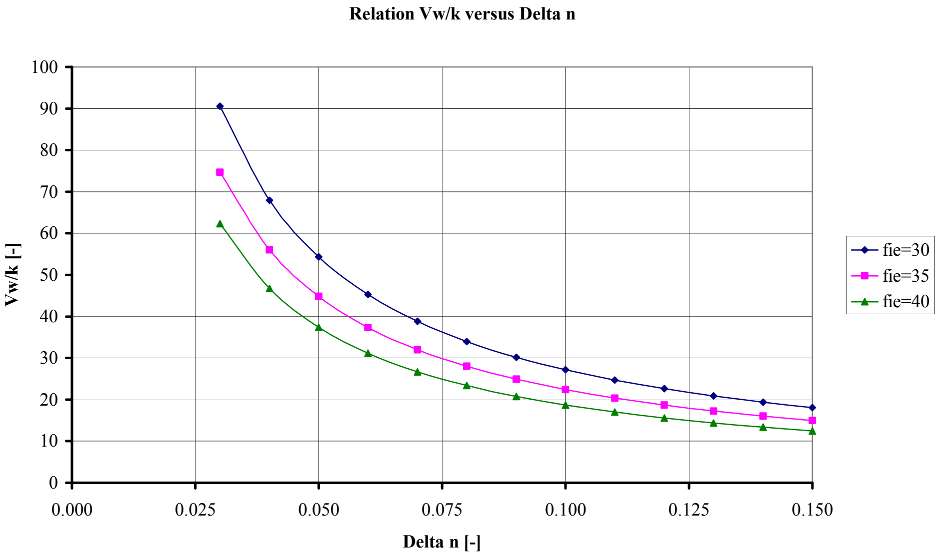
$$v \cdot ds \cdot dt = -v_w dt \cdot ds \sin\gamma \cdot \Delta n$$

Substituted in the above equation gives:

$$v_w = \frac{\rho_k - \rho_w}{\rho_w} \cdot \frac{k}{\Delta n} \cdot \left(\cot\phi - \cot\gamma\right)$$

The results of this formula are shown in the graph below.

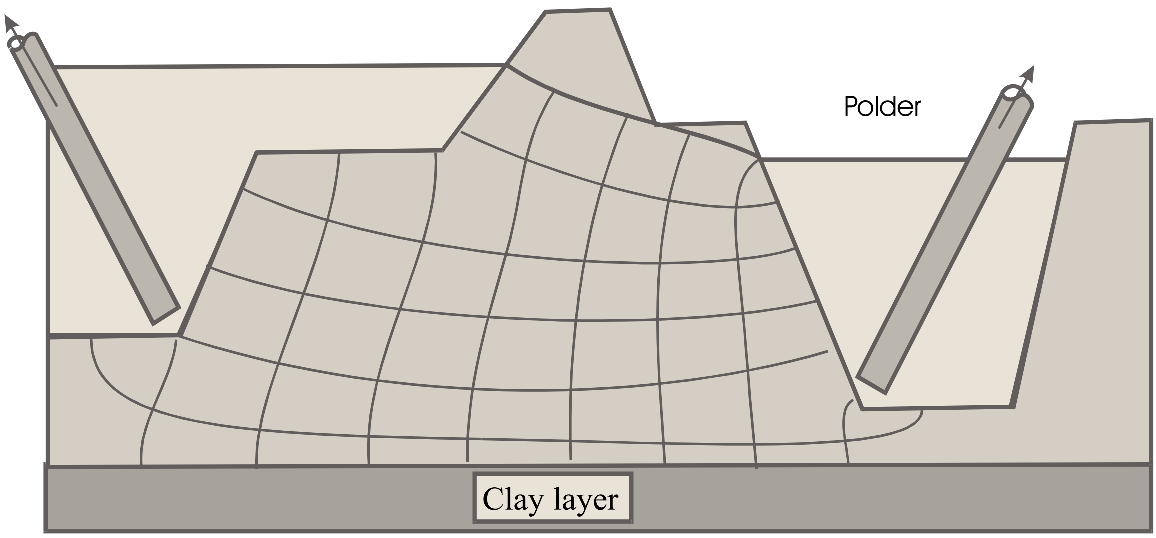




Model tests have showed that v_w equals 20 to 40 times the permeability, which corresponds with for large Δn:

3. Influence groundwater flow on the wall velocity

From the consideration above it appears that the wall velocity can be influenced strongly by a groundwater flow. Such a situation can occur in the neighbourhood of polders. Depending if dredging takes place inside or outside the polder the breaching process is better or worse. The total flow or seepage increases by the in- or outflow v_i.



Groundwater flow from to the polder

Equilibrium of slope:
v= Seepage due to dilatancy

$$\frac{v}{k} = -\frac{\rho_p - \rho_w}{\rho_w}\left(\sin\gamma \cot\phi - \cos\gamma\right)$$

Inflow due to dilatancy:

$$Q = v \cdot ds \cdot dt = -v_w dt \cdot ds \sin\gamma \cdot \Delta n$$

$$v = -v_w \sin\gamma \cdot \Delta n$$

Inflow due to groundwater flow: $\pm v_i$

Total seepage; $v \pm v_i = -v_w \sin\gamma \cdot \Delta n \pm v_i$

Equilibrium of slope:

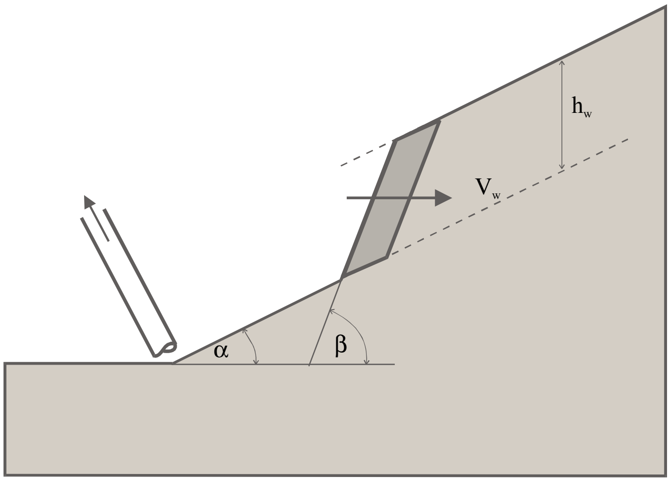
$$v \pm v_i = -v_w \sin\gamma \cdot \Delta n \pm v_i = -\frac{\rho_p - \rho_w}{\rho_w}\left(\sin\gamma \cot\phi - \cos\gamma\right) \cdot k$$

The wall velocity becomes:

$$v_w = \frac{\rho_p - \rho_w}{\rho_w} \cdot \frac{k}{\Delta n}\left(\cot\phi - \cot\gamma\right) \pm \frac{v_i}{\sin\gamma \cdot \Delta n}$$

4. The sand production

Every time when the suction tube is lowered or moved forward causes a distortion and a result a movement of little wall or active bank running away from the suction tube.
The sand production per unit of width of a wall or active bank with a height h_W is $q_w = h_w v_w$ and independent of slope angle β of the wall.
So the sand production of the total slope is $\sum q_w W = \sum h_w v_w W$ in which W is the width of the slope.

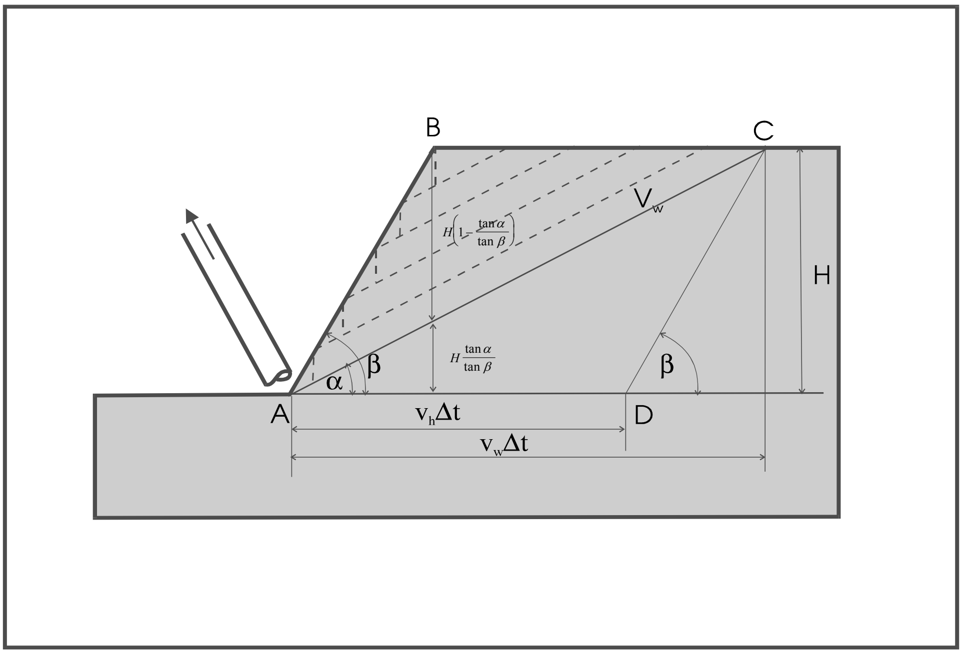


5. The two-dimensional suction process.

Moving the suction tube forward with a constant speed v_h causes a continuous distortion and therefore wall with a infinitive height.
If, in the figure below, the suction tube is moved forward from A to D in a time interval Δt the slope AB has moved to CD and the wall from the last distortion from A to C

This result in the following condition:

$$v_w \Delta t = \frac{H}{\tan\alpha} \text{ en } v_h \Delta t = v_w \Delta t - \frac{H}{\tan\beta} = \frac{H}{\tan\alpha} - \frac{H}{\tan\beta}$$



Elimination of Δt gives:

$$v_h = v_w\left(1 - \frac{\tan\alpha}{\tan\beta}\right)$$

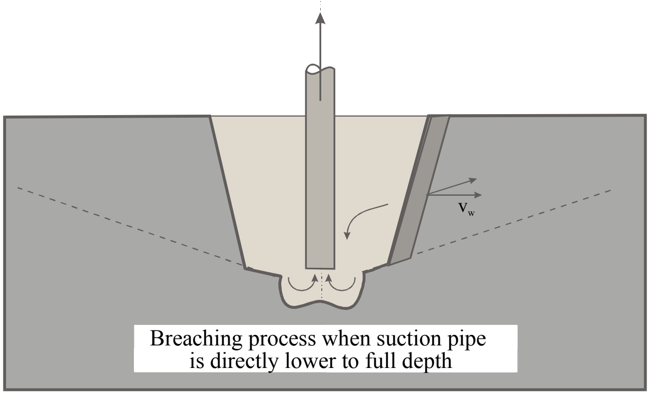
As stated above this should be equal to total production of the slope $v_h HW$.
Production van all walls:

$$Q_{prod} = \sum h_w \cdot v_w \cdot W = v_h \cdot HW = v_w\left(1 - \frac{\tan\alpha}{\tan\beta}\right)HW$$

By Verbeek en Biemond [1, 1966] breach production are derived by this theory for different movements of the suction tube.

5.1. Suction mouth brought directly to full depth.

As said earlier when a suction tube is directly lowered to the full depth of a sand layer, a cylindrical hole is created around the suction tube with almost vertical slopes. When time passes these vertical slopes move away radial from the suction tube with a speed v_w while the sand flows over a certain slope to the suction mouth. The slope angle caused by this phenomena is equal or smaller than the angle of internal friction of the sand. The higher the breach the more material will flow downwards causing a more gentle slope.



Breaching process when suction pipe is directly lower to full depth

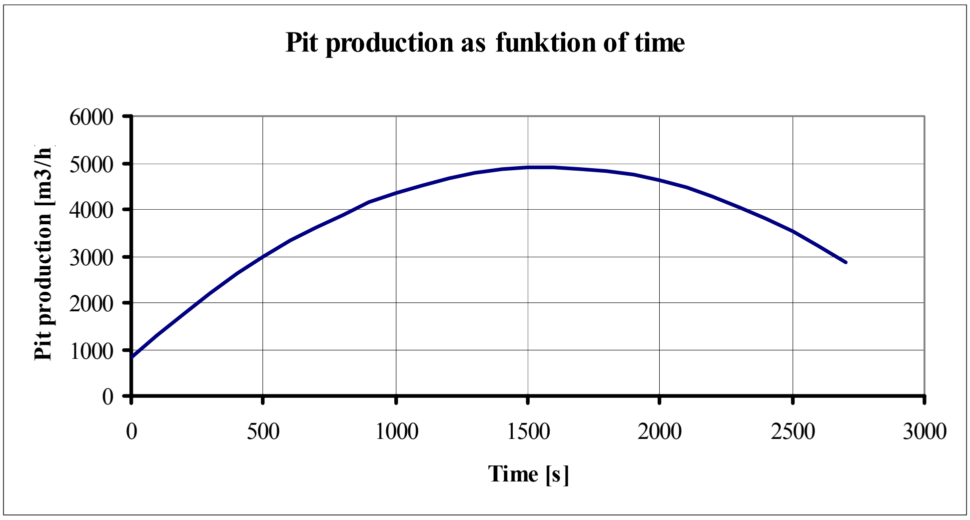
If the radius of the suction tube is $R_{0,}$ the radius of the suction pit will increase with time according to: $R = (R_0 + v_w t)$

The height of the wall is: $h_w = H - (R_0 + v_w t)\tan\alpha$, in which α is the angle of generated by this process.

The production as function of time is now:

$$Q_{prod} = \left[H - (R_0 + v_w t)\tan\alpha\right]\cdot\pi\cdot 2(R_0 + v_w t)v_w$$

The figure below shows the production as function of time when a 0.75 m suction tube is 10 m brought in a sans with a wall speed of 5mm/s and a slope angle at rest of 30°



5.2. Vertical movement of the suction tube (Dredging of cone shaped pits)

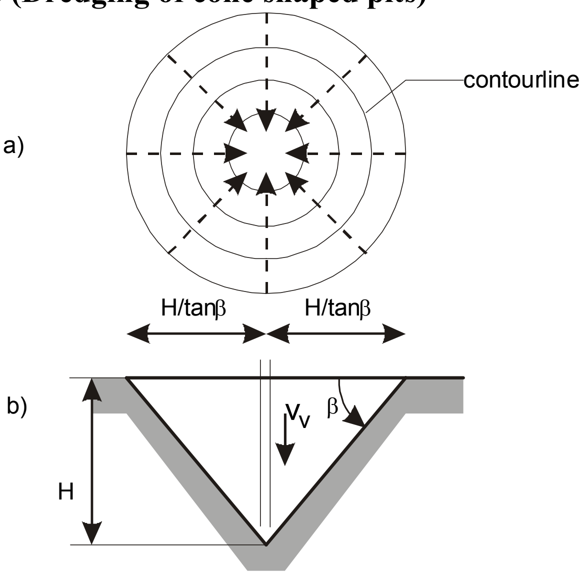
If the vertical speed of the suction tube is v_v the slope will move in the horizontal direction with a speed:

$$v_h = \frac{v_v}{\tan\beta} = v_w\left(1 - \frac{\tan\alpha}{\tan\beta}\right)$$

$$\Rightarrow\ v_v = v_w(\tan\beta - \tan\alpha)$$

Defining the depth of the pit as H=f(t) then:

$$\frac{dh}{dt} = v_v = v_w(\tan\beta - \tan\alpha)$$



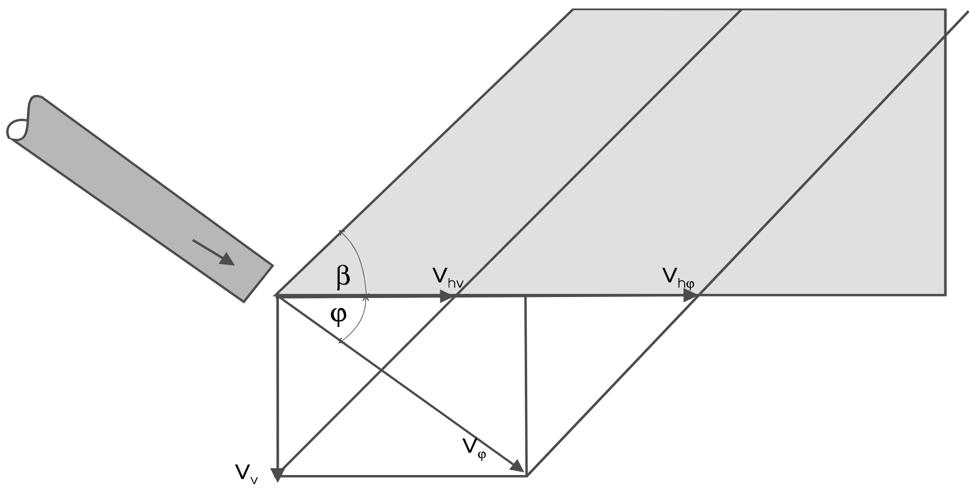
The volume of the pit is:

$$V(t)=\frac{\pi}{3}\frac{H^3}{\tan^2\beta}$$

The production of the pit can now be written as:

$$Q(H)=\frac{dV(t)}{dt}=\frac{dV(t))}{dH}\frac{dH}{dt}=\frac{\pi}{\tan^2\beta}H^2 v_v=\frac{\pi}{\tan^2\beta}H^2\left(\tan\beta-\tan\alpha\right)v_w$$

5.3. The movements of the suction tube under an angle φ



Moving the suction tube downwards with an angle φ>α the slope will move in the horizontal direction according to:

$$v_h=\frac{v_v}{\tan\varphi}+\frac{v_v}{\tan\beta}=v_v\left(\frac{1}{\tan\varphi}+\frac{1}{\tan\beta}\right)=v_w\left(1-\frac{\tan\alpha}{\tan\beta}\right)$$

Hence:

$$v_v=v_w\left(\frac{1-\dfrac{\tan\alpha}{\tan\beta}}{\left(\dfrac{1}{\tan\varphi}+\dfrac{1}{\tan\beta}\right)}\right)=v_w\left(\frac{\dfrac{\tan\beta-\tan\alpha}{\tan\beta}}{\dfrac{\tan\beta+\tan\varphi}{\tan\varphi\tan\beta}}\right)=v_w\left(\frac{\tan\beta-\tan\alpha}{\tan\beta+\tan\varphi}\right)\tan\varphi$$

$$\frac{\dfrac{v_v}{v_w}}{\tan\varphi}=\left(\frac{\tan\beta-\tan\alpha}{\tan\beta+\tan\varphi}\right)\Rightarrow\tan\beta=\frac{\tan\alpha+\dfrac{v_v}{v_w}}{\tan\varphi-\dfrac{v_v}{v_w}}\tan\varphi$$

For the slope behind the suction tube $\varphi' = \pi - \varphi$, which results in

$$\tan\delta = \frac{\tan\alpha + \dfrac{v_v}{v_w}}{\tan(\pi-\varphi) - \dfrac{v_v}{v_w}}\tan(\pi-\varphi) = \frac{\tan\alpha + \dfrac{v_v}{v_w}}{\tan\varphi + \dfrac{v_v}{v_w}}\tan\varphi$$

The base area of the pit has a diameter of:

$$D = H\left[\frac{1}{\dfrac{\tan\alpha + \frac{v_v}{v_w}}{\tan\varphi - \frac{v_v}{v_w}}\tan\varphi} + \frac{1}{\dfrac{\tan\alpha + \frac{v_v}{v_w}}{\tan\varphi + \frac{v_v}{v_w}}\tan\varphi}\right] = H\frac{\left[\tan\varphi - \frac{v_v}{v_w} + \tan\varphi + \frac{v_v}{v_w}\right]}{\left[\tan\alpha + \frac{v_v}{v_w}\right]\tan\varphi} = \frac{2H}{\tan\alpha + \frac{v_v}{v_w}}$$

The surface area is:

$$A = \frac{\pi D^2}{4} = \frac{\pi H^2}{\left[\tan\alpha + \dfrac{v_v}{v_w}\right]^2}$$

The volume of the pit:

$$V(t) = \frac{AH}{3} = \frac{\pi H^3}{\left[\tan\alpha + \dfrac{v_v}{v_w}\right]^2}$$

$$P(H) = \frac{dV(t)}{dt} = \frac{dV(t))}{dH}\frac{dH}{dt} = \frac{\pi}{\tan^2\beta}H^2 v_v = \frac{\pi}{3}\frac{H^2}{\left[\tan\alpha + \dfrac{v_v}{v_w}\right]^2}v_v$$

With:

$$v_v = v_w\left(\frac{\tan\beta - \tan\alpha}{\tan\beta + \tan\varphi}\right)\tan\varphi$$

This results in:

$$P(H)=\frac{\pi}{3}\frac{H^2}{\left[\tan\alpha+\frac{v_v}{v_w}\right]^2}v_v=\frac{\pi}{3}\frac{H^2\left(\frac{\tan\beta-\tan\alpha}{\tan\beta+\tan\varphi}\right)\tan\varphi}{\left[\tan\alpha+\frac{\tan\beta-\tan\alpha}{\tan\beta+\tan\varphi}\tan\varphi\right]^2}v_w$$

In the above theory it is assumed that the base area of the pit is a circle. However, this is questionable. According to the theory the slope angle in a certain direction has to correspond to the horizontal speed of the suction tube in that direction
The slope angle in the direction of the movement is:

$$\tan\beta=\frac{\tan\alpha+\frac{v_v}{v_w}}{\tan\varphi-\frac{v_v}{v_w}}\tan\varphi$$

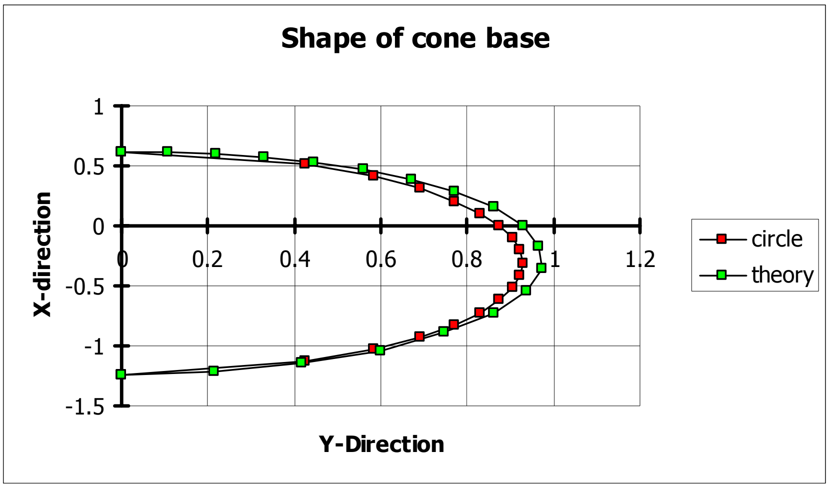
Further $\tan\varphi=\frac{v_v}{v_h}$, so this equation can be written as:

$$\tan\beta=\frac{\tan\alpha+\frac{v_v}{v_w}}{\frac{v_v}{v_h}-\frac{v_v}{v_w}}\cdot\frac{v_v}{v_h}$$

The horizontal of the slope in any direction can be written as: $v_h=\widehat{v}_h\cos\gamma$, in which γ corresponds with the horizontal movement of the suction tube.
So the slope angle in the direction γ is:

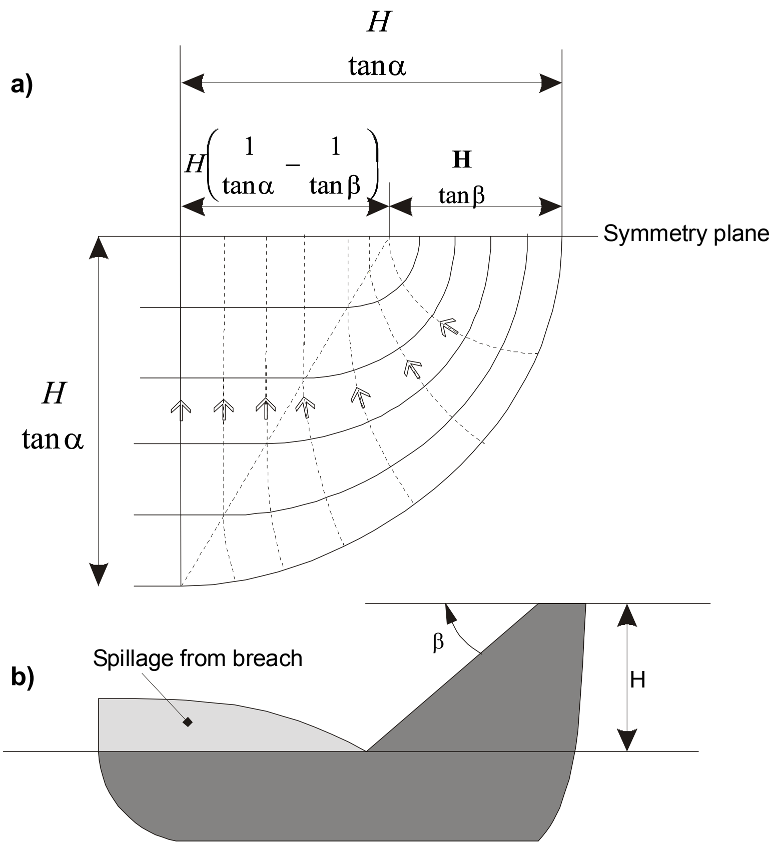
$$\tan\beta=\frac{\tan\alpha+\frac{v_v}{v_w}}{\frac{v_v}{\widehat{v}_h\cos\gamma}-\frac{v_v}{v_w}}\frac{v_v}{\widehat{v}_h\cos\gamma}$$

With these angles the shape of the base of the pit can be calculated as shoen in the figure below together with circle from the original theory.



The figure shows clearly that the base area according to the last theory is bigger. The higher the vertical movement of the suction tube the more the base area is approximated by the circle.
The difference is only of interest from scientific view point!

5.4. A horizontal movement of the suction mouth



When moving the suction mouth horizontal with a speed v_h and a width W the production will be equal to:

$$P = W\frac{H}{2}v_h$$

with $W = \dfrac{2H}{\tan\alpha}$ en $v_h = v_w\left(1 - \dfrac{\tan\alpha}{\tan\beta}\right)$ this results in

$$P = \frac{H^2 v_h}{\tan\alpha} = H^2 v_w\left\{\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right\}$$

If the angle of the side slopes are equal to α, then the production is:

$$P = W\frac{H}{2}v_h = v_h\frac{H^2}{\tan\alpha}$$

For the slope perpendicular on the movement of the suction pipe is the velocity of the slope zero. That means that after the suction has passed this point no distortions on the slope will take place caused by the movement of the suction pipe. As a result of this the angle of the slope finally be the angle of dynamic response α.
The time necessary to reach this equilibrium situation is determined by the last distortion to reach the topside of the face and is therefore:

$$t = \frac{H}{v_w\tan\alpha}$$

In the same time the suction pipe is move forward over a distance $D = v_h t = \dfrac{v_h H}{v_w\tan\alpha}$.

As a result of this a part of the down flowing sand will not reach the suction mouth anymore, but will settle behind it.

So the real production is lower than calculated with the formulae:

$$P_h = v_h\frac{H^2}{\tan\alpha}$$

The question is now if it is possible to calculate the amount of spillage

If it is assumed that the final width of the trench is set to b and that a distortion leaving at point O under an angle γ will just reach the top of the face at T. (See figure below)

Along the BT the slope has reach its equilibrium, so no spillage will occur along the line OB, but only along AB

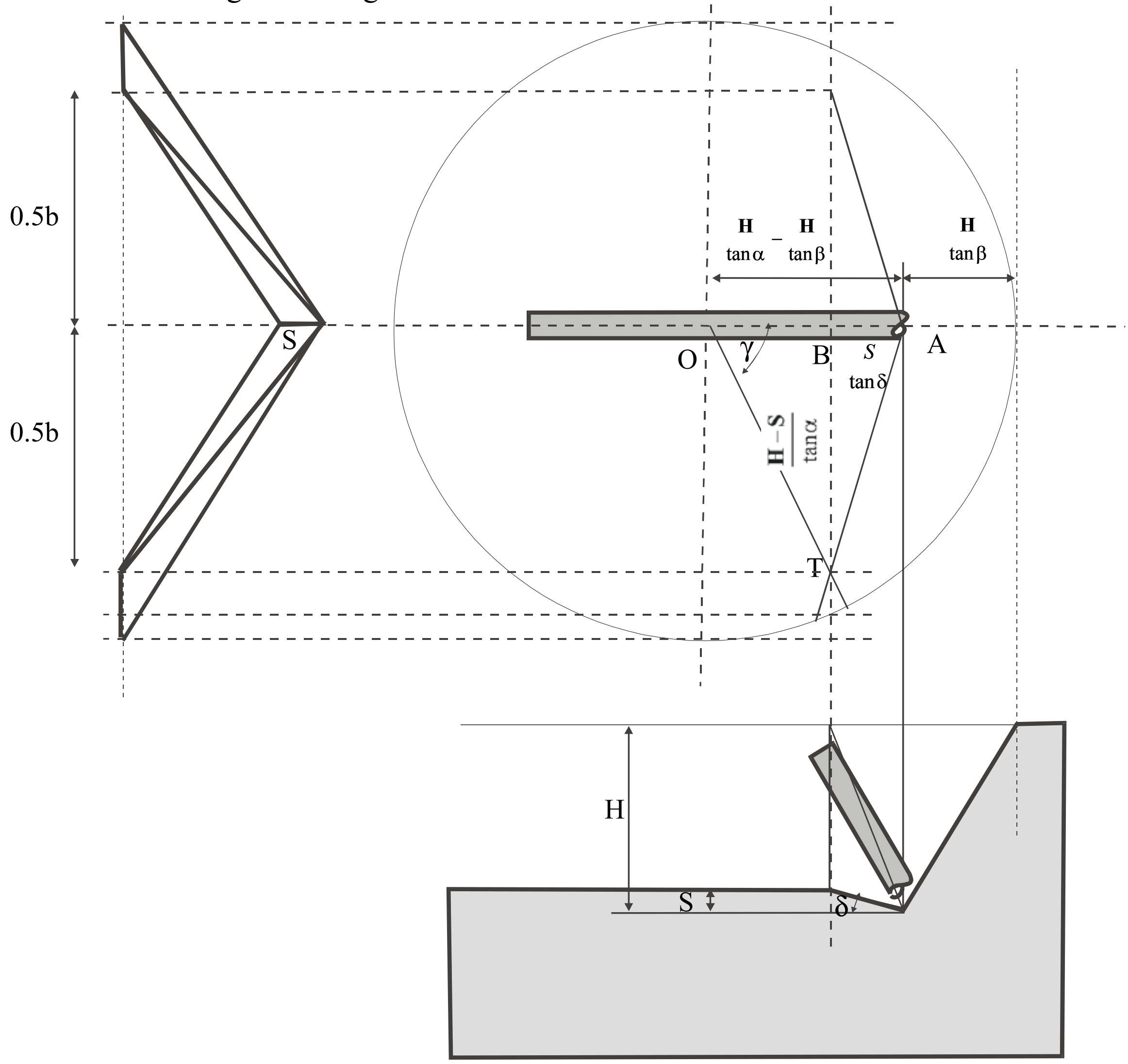
The following production balance can be set up:

$$\frac{H-S}{\tan\alpha} H v_h - \frac{2S}{2} \frac{S}{\tan\delta} v_w = \frac{(H-S)^2}{\tan\alpha} v_h$$

with:

Symbol	Declaration	Dimension
H	Maximum pit depth	M
S	Height of spillage	M
v_h	Horizontal velocity suction mouth	m/s
V_w	Distortion (Wall) velocity	m/s
A	Minimum slope angle=< angle of internal friction	°

Note: ½S is the average wall height over the area considered.



This leads to:

$$\frac{H^2 - HS}{\tan\alpha} - \frac{S^2}{\tan\delta}\frac{v_w}{v_h} = \frac{H^2 - 2HS + S^2}{\tan\alpha}$$

$$H^2 - HS - S^2\frac{\tan\alpha}{\tan\delta}\frac{v_w}{v_h} = H^2 - 2HS + S^2$$

$$S = 0 \;\; and \;\; S = \frac{H}{1 + \frac{\tan\alpha}{\tan\delta}\frac{v_w}{v_h}}$$

The relation between $\tan\alpha$ and $\tan\delta$ follows from the condition that the time taken by last distortion to move from point B to point T equals the time t for moving the suction tube from point B to A.

$$\frac{S}{v_h \tan\delta} = \frac{H - S}{v_w \tan\alpha}$$

or

$$\frac{\tan\delta}{\tan\alpha} = \frac{v_w}{v_h}\frac{S}{(H - S)}$$

In the model tests executed in the dredging laboratory of the Delft University of Technology it appeared that $\frac{\tan\delta}{\tan\alpha} = \frac{1}{4.77} = 0.21$.

So δ is much smaller than the angle of repose α, caused by the sand flow from the slopes. Therefore it is expected that this relation is not constant but will depend on the type of sand and breach height.

The production can be written as:

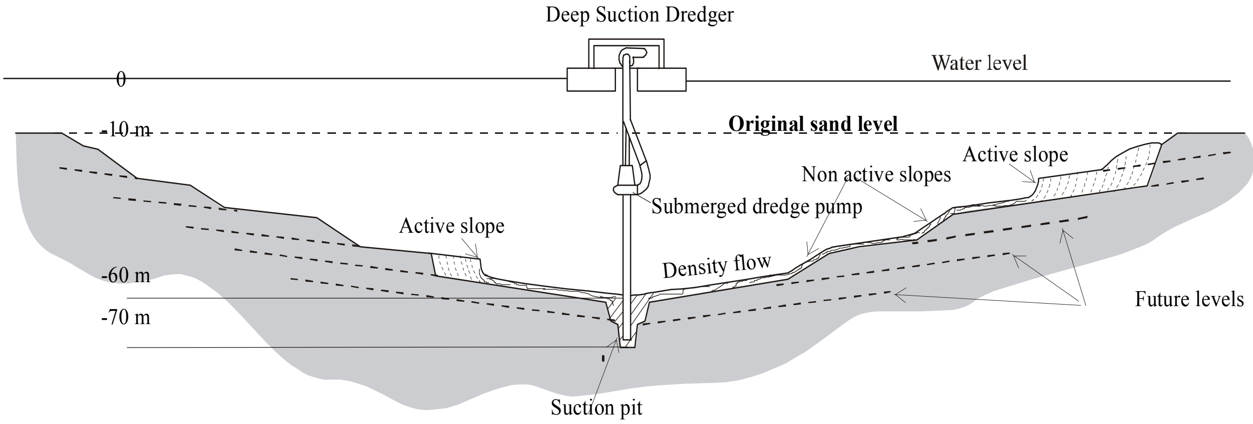
$$Q = \frac{(H - S)^2}{\tan\alpha} v_h = \frac{\left[H - \frac{H}{1 + \frac{\tan\alpha}{\tan\delta}\frac{v_w}{v_h}} \right]^2}{\tan\alpha} v_h$$

$$Q = \frac{H^2}{\tan\alpha}\left[1 - \frac{1}{1 + \frac{\tan\alpha}{\tan\delta}\frac{v_w}{v_h}} \right]^2 v_h = \frac{H^2}{\tan\alpha}\left[\frac{v_w}{\frac{\tan\alpha}{\tan\delta} v_h + v_w} \right]^2 v_h$$

From this equation follows that the maximum production is reach at v_h=v_w or when the slope β=90°.

6. Practical considerations

Under prototype circumstances with breach height of 5 meters or more, the instabilities are of a larger scale and called active fronts. As a result of the bigger volumes involved a turbulent density current starts accelerates along the slope. At the base of the breach a very gentle slope occurs, much smaller than the natural slope (De Koning, 1970) It is even possible that sedimentation will take place at the base of the breach. Instead of a cone shaped pit, the shape is more or less basin shaped.

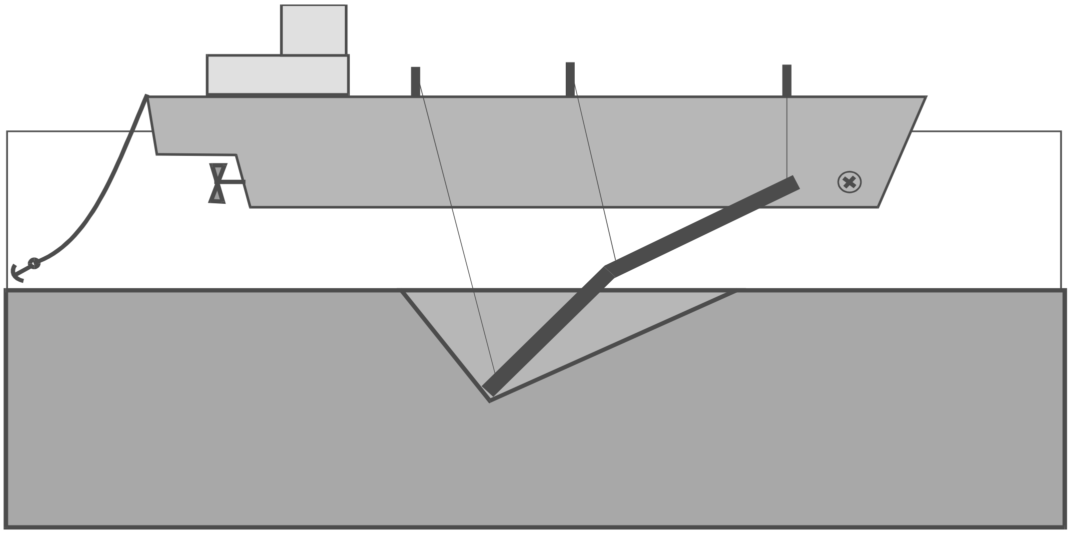


Almost all plain suction dredgers are equipped with water jets to improve the breaching process when necessary and/or to support the mixture forming process near the suction mouth in order to dredge the sand with the required density.
Although , in principle, the complete breaching process can be described with the existing erosion and sedimentation theories together with instability calculations of the slope, it will be clear that due to the variation of the soil, the existence of layers of cohesive material etc, the accuracy of the calculation is limited.
Borrow pits from plain suction dredgers are therefore very irregular in shape instead of having the shape of a cone. Due to the variation of the soil and the size of the pit some segments of the pit may breach quite different than others.

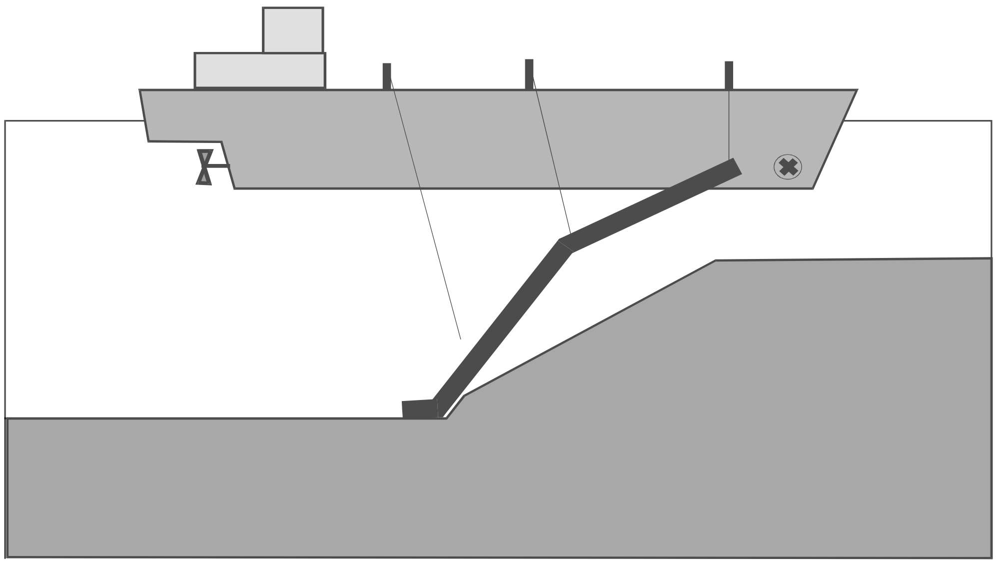
Dredging with plain suction dredgers is more attractive when the production of the breach exceeds the production of the pumps of the dredger. When this is not the case, the breaching process has to be activated by powerful water jets or when the breach height allows it, a cutter suction dredger will be used to reach the higher production level.
The latter has the advantage that more sand can be borrowed from the pit due to a more accurate dredging method

At sea a trailer suction hopper dredger (TSHD) can be used as either in its original mode or in the plain suction modes
The latter can be done in three different ways:

- As a plain suction dredger; The TSHD lays on its stern anchor to keep the suction pipes, from which the dragheads are removed, in the pit, assisted by the bow propellers

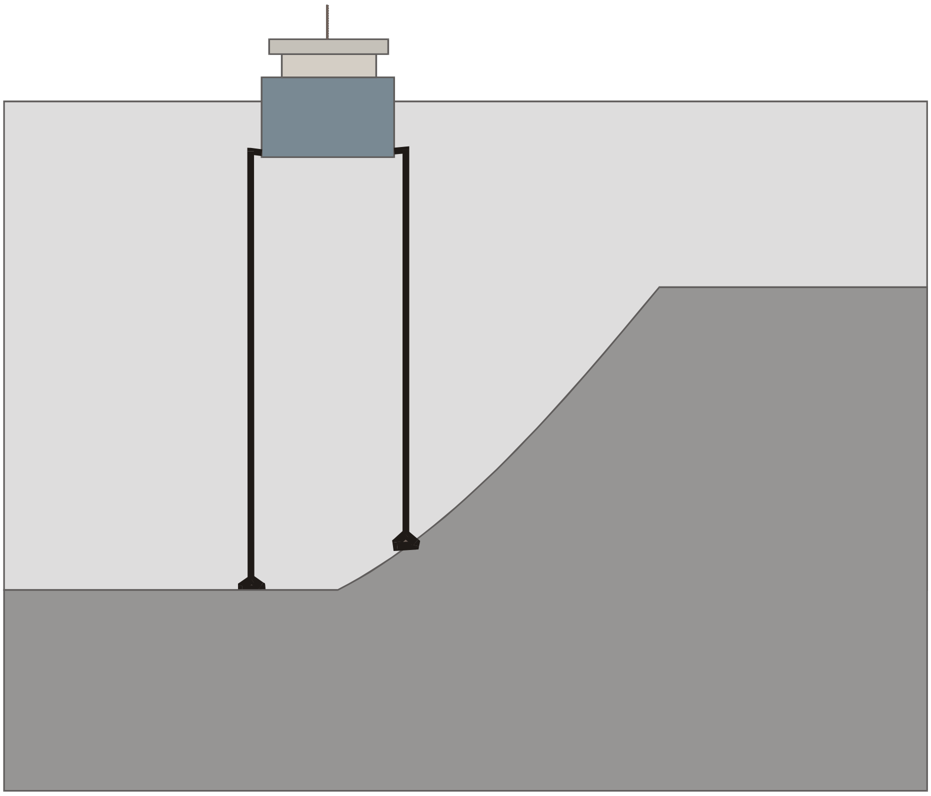


- Dredging perpendicular to the breach.
  When the material to be dredge breaches well dredging perpendicular to the breach with slow trailing speeds depending on the breaching production, which is in the order of 10 to 20 m per hour. The next load is started besides the previous one to shift the total face of the breach regular



It will be clear that the method depends on the slope of the breach and the position of the suction tubes. Is this method not possible dredging can be done in the next mode.

- The 2D breaching process mode.
  First a trench is made to the base of the sand layer or the required depth. After which he TSHD is now dredging along one side of the base of the breach. This process is comparable with the 2D breaching process with a small horizontal speed perpendicular to the slope. The advantage of this method that indeed the sand particles coming down the slope due to the breaching process will settle on the base of the slope but on the other hand will have a less situ density and therefore more easy is to dredge. Besides this small cohesive layers will now be mixed up with the sand.



The 2 D suction mode

7. References

7.1. Biemond Jr, C. & Verbeek, A.R.M., *The dredging Process (in Dutch),* Stevin Dredging, 1969

7.2. Koning, J. de, *Neue Erkentnissen beim Gewinnen und Tranport von Sand in Spülproject Venserpolder*, VDI berichten no. 158, page 185-193, 1970

7.3. Breusers, H.N.C., *Suction of sand ,* Bulletin International Association of Engineering Geology, 1974

7.4. Breusers, H.N.C., *Hydraulic excavation of sand ,* Proceedings International Course in Modern dreding, June 1977, The Haque.

7.5. Verruijt, A., *Offshore Soil Mechanics*, Chapter 4,

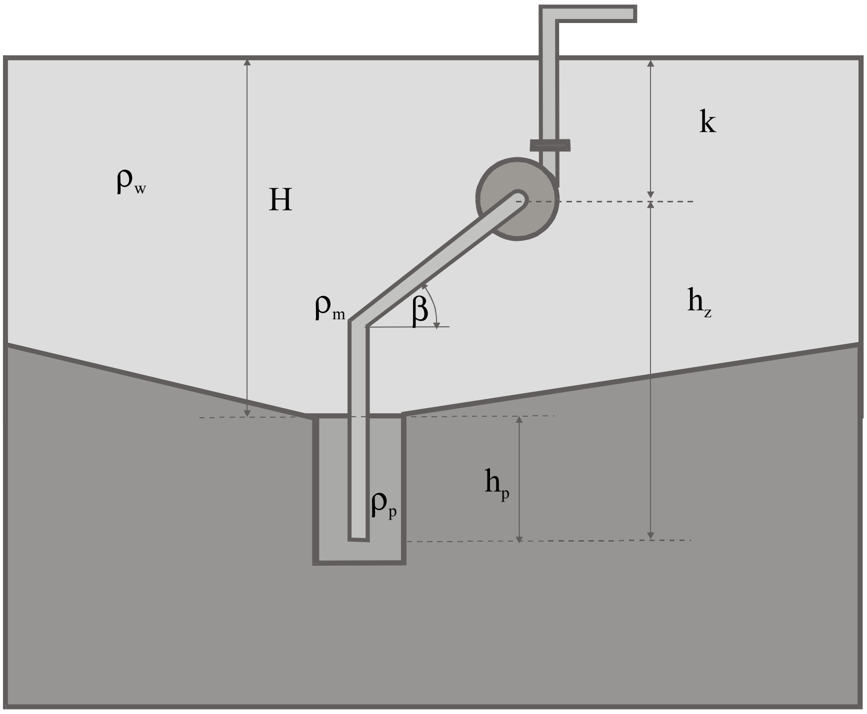
7.6. Berg, J.H. van den, Gelder, A. van, Mastbergen, D.R., *The importance of breaching as a mechanism of sub-aqueous slope failure in fine sand,* Sedimentology (2002) 49, page 81-95

8. ENCLOSURE

8.1. The vacuum formulae for plain suction dredging

The positioning of the dredge pump far below the water level has contributed to the success of the plain suction dredger, which will be explained below.

The maximum production when dredging sand from significant depth is mostly determined by the allowable pressure at the suction site of the dredge pump and the depth of the pump below the water level.



Scheme for vacuum formulae

When the density of the water id ρ_w , the density of the mixture in the suction pipe ρ_m en the mixture in the pit ρ_p then the following force balance can be set up with a reference to the atmospheric pressure:

$$\rho_w gH + \rho_p gh_p - p_{pomp} = \rho_m gh_z + \frac{1}{2}\rho_m v^2\left(1 + \zeta + \lambda\frac{L}{D}\right)$$

with

ρ_w = density water [kg/m^3]

ρ_p = density suspended sand in the pit [kg/m^3]

ρ_m = mixture dichtheid in the suction pipe [kg/m^3]

H = water depth [m]

h_p = depth of the pit [m]

h_z = zuighoogte [m]

p_{pomp} = druk voor de pomp [N/m^2]

v = mixture velocity in the suction pipe [m/s]

ζ = entree losses coefficient[-]

λ = Darcy Weisbach coefficient [-]

L = Total lengt suction pipe [m]

D = diameter suction pipe [m]

The above equation can be rewritten as below to calculate te mixture density:

$$\rho_m = \frac{\rho_w g H + \rho_p g h_p - p_{pomp}}{g h_z + \frac{1}{2} v^2 \left(1 + \zeta + \lambda \frac{L}{D}\right)}$$

With $h_z = H + h_p - k$ gives:

$$\rho_m = \frac{\rho_w g H + \rho_p g h_p - p_{pomp}}{g\left(H + h_p - k\right) + \frac{1}{2} v^2 \left(1 + \zeta + \lambda \frac{L}{D}\right)}$$

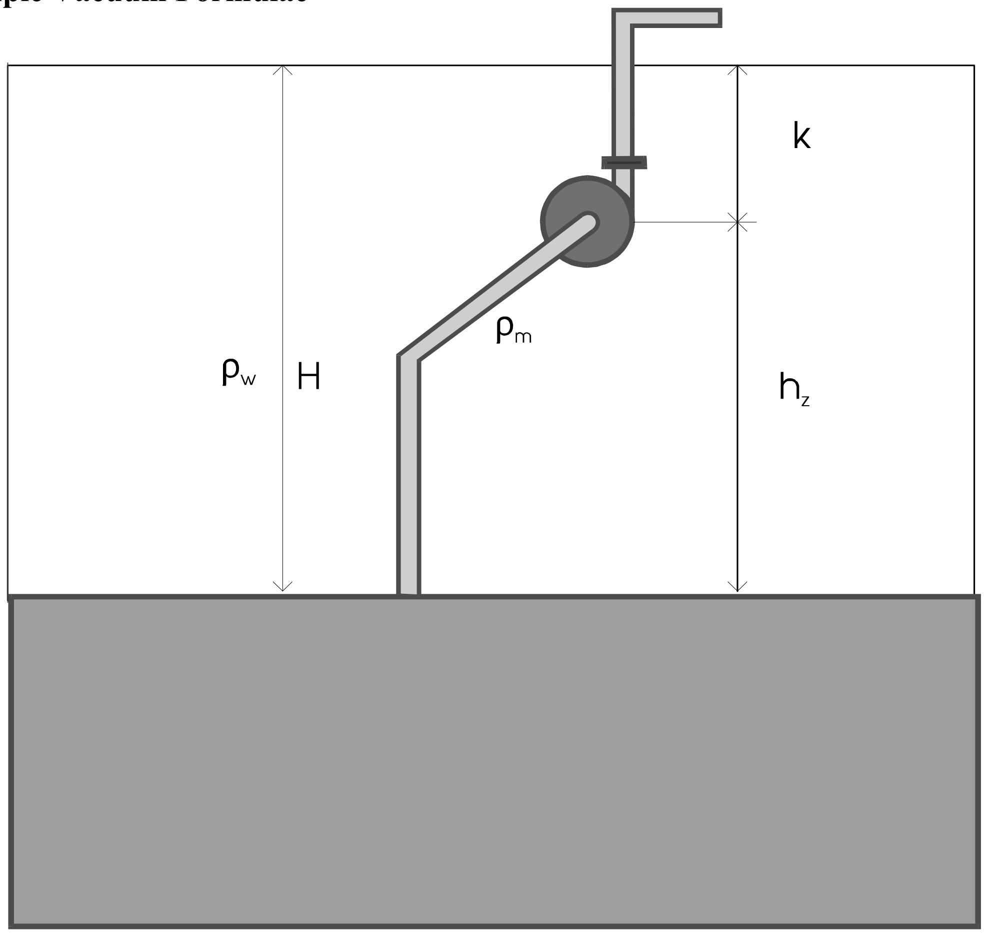
This formulae shows clearly that the maximum density that can be dredged increases with the distance from the pump centre to the water level

It can be proven, (see chapter 8.2), that there is a optimum mixture velocity for which the dredged production is maximum under the condition that this process is decisive.
In the above consideration it is assume that the flow is pseudo homogeneous. Is this not the case the consideration about the optimum velocity is different

NOTE;

In see chapter 8.2 it is assumed that $h_p = 0$, $\xi = 1 + \zeta + \lambda \frac{L}{D}$ en p_{pomp}=-Vac

8.2. **Simple Vacuum Formulae**



The vacuum formulae is a force balance over the suction line.
If:

H	= water depth in m
h_Z	= suction depth in m
Vac	= mean decisive vacuum of dredge pump in N/m²
v	= mean mixture velocity in m
k	= depth of pump below water level in m
ξ	= coefficient of head loss
ρ_{water}	= density of water in kg/m³
$\rho_{mixture}$	= density of mixture in kg/m³

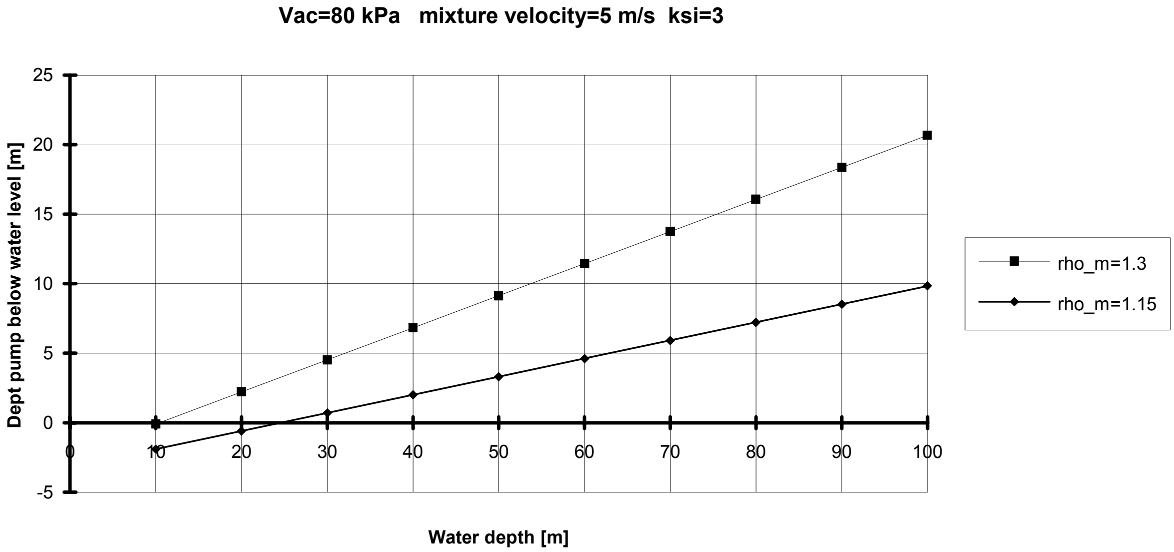
Then

$$\rho_{water} gH + Vac = \rho_{mixture} gh_z + \xi \frac{1}{2} \rho_{mixture} v^2 = \rho_{mixture} g(H-k) + \xi \frac{1}{2} \rho_{mixture} v^2 \qquad [\,1\,]$$

Rewriting gives :

$$k = \frac{gH\left(\rho_{mengsel} - \rho_{water}\right) - Vac + \xi \frac{1}{2}\rho_{mengsel} v^2}{\rho_{mengsel} g} \qquad [\ 2\ ]$$

This expression gives the relation between the required depth of the dredge pump below the water level as function of the water depth, for a given mixture density and mixture velocity and the maximum mean available vacuum of the dredgepump.



Equation [1] can also written as: $\rho_{\text{mixture}} = \dfrac{\rho_{\text{water}} gH + Vac}{(H - k)g + .5\xi v^2}$

[3]

The production in m³ of the system is $Q_s = C_{vd} Q_m = C_{vd} vA$ [4]

Q_s = Capacity in m³/s
C_{vd} = transport concentration
Q_m = Capacity in m³/s
A = pipe cross section in m²

C_{vd} can be written as $C_{vd} = \dfrac{\rho_{\text{mixture}} - \rho_{\text{water}}}{\rho_{\text{grains}} - \rho_{\text{water}}} = \dfrac{\rho_{\text{mixture}}}{S - 1} - \dfrac{1}{S - 1}$ [5]

Substituting [3] and [5] in [4] gives: $Q_s = \dfrac{vA}{S-1}\left[\dfrac{\rho_{\text{water}} gH + Vac}{(H-k)g + .5\xi v^2} - 1\right]$

[6]

Simplifying this function as: $$Q_s = B_1 v \left[\frac{B_2}{B_3 + B_4 v^2} - 1 \right]$$

in which:
$$B_1 = \frac{A}{S-1}$$
$$B_2 = \rho_{water} gH + Vac$$
$$B_3 = (H-k)g$$
$$B_4 = \frac{\xi}{2}$$

The first derivative to v is:
$$\frac{dQ_s}{dv} = B_1 \left[\frac{B_2}{B_3 + B_4 v^2} - 1 \right] + B_1 v \left[\frac{-B_2 2B_4 v}{\left(B_3 + B_4 v^2\right)^2} \right]$$
$$\frac{dQ_s}{dv} = \frac{B_1 B_2 \left(B_3 + B_4 v^2\right) - B_1 \left(B_3 + B_4 v^2\right)^2 - 2B_1 B_2 B_4 v^2}{\left(B_3 + B_4 v^2\right)^2}$$
[7]

This function has a maximum for: $B_1 B_2 \left(B_3 + B_4 v^2\right) - B_1 \left(B_3 + B_4 v^2\right)^2 - 2B_1 B_2 B_4 v^2 = 0$
[8]

Dividing by B1 and reorganised gives: $B_4^2 v^4 + \left(2B_3 B_4 + B_2 B_4\right) v^2 + B_3^2 - B_2 B_3 = 0$ [9]

simplifying gives: $D_4 v^4 + D_2 v^2 + D_0 = 0$ [10]

in which:
$$D_4 = B_4^2$$
$$D_2 = B_4\left(2B_3 + B_2\right)$$
$$D_0 = B_3^2 - B_2 B_3$$

with the solution: $$v = \sqrt{\left(\frac{-D_2 \pm \sqrt{D_2^2 - 4D_4 D_0}}{2D_4} \right)}$$
[11]

Only the positive sign gives a real solution so: $$v = \sqrt{\left(\frac{-D_2 + \sqrt{D_2^2 - 4D_4 D_0}}{2D_4} \right)}$$ [12]

Substituting the values for D_0, D_2, and D_4 gives: $v = \sqrt{\left(\frac{-2B_3 - B_2 + \sqrt{B_2^2 + 8B_3B_2}}{2B_4}\right)}$ [13]

Realise that v is independent of B_1 and so of the pipe diameter.